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Sector 15, Noida

CLASS 12 - MATHEMATICS

Paper 2

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ then A is a **[1]**

   a) skew-symmetric matrix b) symmetric matrix

   c) none of these d) diagonal matrix

2. $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$ is equal to **[1]**

   a) a positive real number b) a negative real number

   c) 0 d) None of these

3. If f (x) = x $\tan^{-1}$ x then f ‘ (1) is equal to **[1]**

   a) None of these b) $\frac{1}{2} - \frac{\pi}{4}$

   c) $\frac{\pi}{4} - \frac{1}{2}$ d) $\frac{\pi}{4} + \frac{1}{2}$

4. If x = at^2 ,y = 2 a t, then $\frac{d^2y}{dx^2}$ is equal to **[1]**

   a) None of these b) 0

   c) $\frac{1}{t^2}$ d) $-\frac{1}{2a\ t^3}$

5. Forming a differential equation representing the given family of curves by eliminating arbitrary constants a and b from $y^2 = a\left(b^2 - x^2\right)$ yields the differential equation **[1]**

a) $yy'' + y'^2 - 1 = 0$ b) $yy'' + y'^2 = 0$

c) $yy'' + y'^2 + 1 = 0$ d) $yy'' - y'^2 + 1 = 0$

6. The values of x which satisfy the trigonometric equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ are : **[1]**

a) ± 2 b) $\pm\frac{1}{2}$

c) $\pm\frac{1}{\sqrt{2}}$ d) $\pm\sqrt{2}$

7. Let X be a random variable assuming values x_1, x_2,....,x_n with probabilities p_1, p_2, ...,pn, respectively such that $p_i \geq 0, \sum_{i=1}^{n} p_i = 1$. Mean of X denoted by μ is defined as **[1]**

a) $\mu = \sum_{i-1}^{n} x_i p_i$ b) $\mu = \sum_{i-1}^{n} p_i$

c) $\mu = \sum_{i-1}^{n} x_i p_{i+1}$ d) $\mu = \sum_{i-1}^{n} x_i$

8. $\int_0^{\pi/2} \frac{1}{1+\tan x} dx$ is equal to **[1]**

a) 1 b) $\frac{3\pi}{2}$

c) π d) $\frac{\pi}{4}$

9. Find the equations of the planes that passes through three points (1, 1, 0), (1, 2, 1), (– 2, 2, – 1) **[1]**

a) 2x + 3y – 3z = 5 b) 3x + 3y – 3z = 5

c) 2x + 5y – 3z = 5 d) 2x + 3y – 7z = 5

10. If two vectors have their corresponding direction cosines equal then the two vectors **[1]**

a) are at an angle of 55° b) are at an angle of 45°

c) are parallel d) are perpendicular

11. Fill in the blanks: **[1]**

A relation R from a set X to a set Y is defined as a _______ of the cartesian product X $\times$ Y.

12. Fill in the blanks: **[1]**

When two coins are tossed simultaneously, the chances of getting atleast one tail is _______.

13. Fill in the blanks: **[1]**

If A is matrix of order m × n and B is a matrix such that AB' and B'A are both defined, then order of matrix B is _______.

14. Fill in the blanks: **[1]**

The indefinite integral of $x^{\frac{1}{4}}$ is _______.

OR

Fill in the blanks:

The indefinite integral of $2x^2 + 3$ is _______.

15. Fill in the blanks: **[1]**

The maximum or minimum value of an objective function is known as _______.

OR

Fill in the blanks:

In a LPP, the objective function is always ________.

16. If A is an invertible matrix of order 3 and |A| = 5, then find | adj A|. **[1]**

17. Find the cartesian equation of the line which passes through the point (-2, 4, - 5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. **[1]**

18. Write the value of the following integral **[1]**

$\int 2 \sin x d x$

OR

Find $\int \frac{d x}{x^2+4 x+8}$.

19. Find the interval in which the function f(x) = x^2e^{-x} is increasing. **[1]**

20. Compute the magnitude of $\vec{b}=2 \hat{i}-7 \hat{j}-3 \hat{k}$ **[1]**

Section B

21. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g(x) = ax + b, then what value should be assigned to a and b. **[2]**

22. Find the second-order derivative of the function x cos x **[2]**

OR

Find $\frac{d y}{d x}$, if $y=\sin ^{-1}\left(\frac{2 x}{1+x^2}\right)$

23. If $\vec{a} . \vec{a}=0$ and $\vec{a} . \vec{b}=0$, then what can be concluded about the vector $\vec{b}$? **[2]**

24. Find the approximate value of f(3.02) where $f(x)=3 x^2+5 x+3$ **[2]**

OR

An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

25. If the direction ratios of a line are 1,1,2 find the direction cosines of the line. **[2]**

26. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1- P(A')(P(B'). **[2]**

Section C

27. If $\tan ^{-1} \frac{x-1}{x-2}+\tan ^{-1} \frac{x+1}{x+2}=\frac{\pi}{4}$,find x **[4]**

28. Prove that $\frac{d}{d x}\left[\frac{x}{2} \sqrt{a^2-x^2}+\frac{a^2}{2} \sin ^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$. **[4]**

OR

Prove that the greatest integer function defined by $f(x)=[x], 0<x<3$ is not differentiable at x = 1 .

29. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10%, respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian? **[4]**

30. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs 100 and that on a bracelet is Rs 300. Formulate on L.P.P. for finding **[4]**

how many of each should be produced daily to maximise the profit? It is being given that at least one of each must be produced.

31. Solve the following differential equation. **[4]**

$\cos^2 x \frac{dy}{dx} + y = \tan x$

OR

Find the general solution: $\frac{dy}{dx} + 3y = e^{-2x}$

32. Evaluate $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}} dx$ **[4]**

Section D

33. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = 0$. Hence find A^{-1}. **[6]**

OR

Find x and y, if 2x + 3y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3x + 2y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

34. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$. **[6]**

35. Using integration, find the area of the region $\{(x, y): x^2 + y^2 \leq 16, x^2 \leq 6y\}$. **[6]**

OR

Using integration, find the area of the region given below:

$\{(x,y) : 0 \leqslant y \leqslant x^2 + 1, 0 \leqslant y \leqslant x + 1, 0 \leqslant x \leqslant 2\}$.

36. Find the coordinate where the line thorough (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7. **[6]**